PARI-GP Reference Card

(PARI-GP version 2.6.1)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	`?`*function*
extended description	`??`*keyword*
list of relevant help topics	`???`*pattern*

Input/Output

previous result, the result before	`%`, `%'`, `%''`, etc.
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	{seq_1`;` seq_2`;`}
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to *val*	`default`$(\{d\},\{val\},\{flag\})$
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set output mode (raw=0, default=1)	`\o` n
set n significant digits	`\p` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up n frames	`dbg_up`$(\{n\})$
examine object o	`dbg_x`(o)

PARI Types & Input Formats

`t_INT`/`t_REAL`. Integers, Reals	$\pm n$, $\pm n.ddd$
`t_INTMOD`. Integers modulo m	`Mod`(n,m)
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field F_q	`ffgen(q)`
`t_COMPLEX`. Complex Numbers	$x+y*$`I`
`t_PADIC`. p-adic Numbers	$x+$`O(p^k)`
`t_QUAD`. Quadratic Numbers	$x+y*$`quadgen`(D)
`t_POLMOD`. Polynomials modulo g	`Mod`(f,g)
`t_POL`. Polynomials	$a*$`x^n+`$\cdots+b$
`t_SER`. Power Series	$f+$`O(x^k)`
`t_QFI`/`t_QFR`. Imag/Real bin. quad. forms	`Qfb`$(a,b,c,\{d\})$
`t_RFRAC`. Rational Functions	f/g
`t_VEC`/`t_COL`. Row/Column Vectors	`[`x,y,z`]`, `[`x,y,z`]~`
`t_MAT`. Matrices	`[`$x,y;z,t;u,v$`]`
`t_LIST`. Lists	`List([`x,y,z`])`
`t_STR`. Strings	`"abc"`

Reserved Variable Names

$\pi=3.14\ldots$, $\gamma=0.57\ldots$, $C=0.91\ldots$	`Pi`, `Euler`, `Catalan`
square root of -1	`I`
big-oh notation	`O`

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Information about an Object

PARI type of object x	`type`(x)
length of x / size of x in memory	`#`x, `sizebyte`(x)
real or p-adic precision of x	`precision`(x), `padicprec`

Operators

basic operations	`+`, `-`, `*`, `/`, `^`
`i=i+1`, `i=i-1`, `i=i*j`, ...	`i++`, `i--`, `i*=j`,...
euclidean quotient, remainder	x`\/`y, x`\`y, x`%`y, `divrem`(x,y)
shift x left or right n bits	x`<<`n, x`>>`n or `shift`$(x,\pm n)$
comparison operators	`<=`, `<`, `>=`, `>`, `==`, `!=`, `===`, `lex`, `cmp`
boolean operators (or, and, not)	`||`, `&&`, `!`
bit operations	`bitand`, `bitneg`, `bitor`, `bitxor`
sign of $x=-1,0,1$	`sign`(x)
maximum/minimum of x and y	`max`, `min`(x,y)
integer or real factorial of x	x`!` or `factorial`(x)
derivative of f w.r.t. x	f`'`
apply differential operator	`diffop`
restore x as a formal variable	x`='`x
simultaneous assignment $x\leftarrow v_1$, $y\leftarrow v_2$	`[x,y] = v`

Select Components

n-th component of x	`component`(x,n)
n-th component of vector/list x	x`[`n`]`
components $a,a+1,\ldots,b$ of vector x	x`[`a`..`b`]`
(m,n)-th component of matrix x	x`[`m`,`n`]`
row m or column n of matrix x	x`[`m`,]`, x`[,`n`]`
numerator/denominator of x	`numerator`(x), `denominator`

Conversions

to vector, matrix, set, list, string	`Col/Vec`,`Mat`,`Set`,`List`,`Str`
create PARI object (x mod y)	`Mod`(x,y)
make x a polynomial of v	`Pol`$(x,\{v\})$
as `Pol`/`Vec`, starting with constant term	`Polrev`, `Vecrev`
make x a power series of v	`Ser`$(x,\{v\})$
string from bytes / from format+args	`Strchr`, `Strprintf`
convert x to simplest possible type	`simplify`(x)
object x with precision n	`precision`(x,n)

Conjugates and Lifts

conjugate of a number x	`conj`(x)
conjugate vector of algebraic number x	`conjvec`(x)
norm of x, product with conjugate	`norm`(x)
square of L^2 norm of vector x	`norml2`(x)
lift of x from Mods	`lift`, `centerlift`(x)

Lists, Sets & Sorting

sort x by k-th component	`vecsort`$(x,\{k\},\{fl=0\})$
min. m of x ($m=x[i]$), max.	`vecmin`$(x,\{\&i\})$, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch`$(x,y,\{f\})$

Sets (= row vector of strings with strictly increasing entries)

intersection of sets x and y	`setintersect`(x,y)
set of elements in x not belonging to y	`setminus`(x,y)
union of sets x and y	`setunion`(x,y)
does y belong to the set x	`setsearch`$(x,y,\{flag\})$
is x a set ?	`setisset`(x)

Lists. create empty list: $L=$ List()

append x to list L	`listput`$(L,x,\{i\})$
remove i-th component from list L	`listpop`$(L,\{i\})$
insert x in list L at position i	`listinsert`(L,x,i)
sort the list L in place	`listsort`$(L,\{flag\})$

Programming

Functions and closures

`fun(vars) = my(local vars);` *seq*
`fun = (vars) -> my(local vars);` *seq*

Control Statements (X: formal parameter in expression *seq*)

eval. *seq* for $a\le X\le b$	`for`$(X=a,b,seq)$
eval. *seq* for X dividing n	`fordiv`(n,X,seq)
eval. *seq* for primes $a\le X\le b$	`forprime`$(X=a,b,seq)$
eval. *seq* for $a\le X\le b$ stepping s	`forstep`$(X=a,b,s,seq)$
multivariable `for`	`forvec`$(X=v,seq)$
loop over partitions of n	`forpart`$(p=n\ seq)$
loop over vectors v, $q(v)\le B$, $q>0$	`forqfvec`(v,q,b,seq)
loop over subgrps H of abelian grp G	`forsubgroup`$(H=G)$
evaluate *seq* until $a\neq 0$	`until`(a,seq)
while $a\neq 0$, evaluate *seq*	`while`(a,seq)
exit n innermost enclosing loops	`break`$(\{n\})$
start new iteration of n-th enclosing loop	`next`$(\{n\})$
return x from current subroutine	`return`$(\{x\})$
raise an exception	`error`$()$
if $a\neq 0$, evaluate seq_1, else seq_2	`if`$(a,\{seq_1\},\{seq_2\})$
try seq_1, evaluate seq_2 on error	`iferr`(seq_1,E,seq_2)
select from v according to f	`select`(f,v)
apply f to all entries in v	`apply`(f,v)

Input/Output

print with/without `\n`, TEX format	`print`, `print1`, `printtex`
formatted printing	`printf`$()$
write *args* to file	`write`, `write1`, `writetex`$(file,args)$
write x in binary format	`writebin`$(file,x)$
read file into GP	`read`$(\{file\})$
read file, return as vector of lines	`readvec`$(\{file\})$
read a string from keyboard	`input`$()$

Interface with User and System

allocates a new stack of s bytes	`allocatemem`$(\{s\})$
alias *old* to *new*	`alias`(new,old)
install function from library	`install`$(f,code,\{gpf\},\{lib\})$
execute system command a	`system`(a)
as above, feed result to GP	`extern`(a)
as above, return GP string	`externstr`(a)
get `$VAR` from environment	`getenv("VAR")`
measure time in ms.	`gettime`$()$
timeout command after s seconds	`alarm`$(s,expr)$

Iterations, Sums & Products

numerical integration	`intnum`$(X=a,b,expr,\{flag\})$
sum *expr* over divisors of n	`sumdiv`$(n,X,expr)$
`sumdiv`, with *expr* multiplicative	`sumdivmult`$(n,X,expr)$
sum $X=a$ to $X=b$, initialized at x	`sum`$(X=a,b,expr,\{x\})$
sum of series *expr*	`suminf`$(X=a,expr)$
sum of alternating/positive series	`sumalt`, `sumpos`
sum of series using `intnum`	`sumnum`
product $a\le X\le b$, initialized at x	`prod`$(X=a,b,expr,\{x\})$
product over primes $a\le X\le b$	`prodeuler`$(X=a,b,expr)$
infinite product $a\le X\le\infty$	`prodinf`$(X=a,expr)$
real root of *expr* between a and b	`solve`$(X=a,b,expr)$

Random Numbers

random integer/prime in $[0,N[$	`random`(N), `randomprime`
get/set random seed	`getrand`, `setrand`(s)

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Vectors & Matrices

dimensions of matrix x	`matsize`(x)
concatenation of x and y	`concat`$(x,\{y\})$
extract components of x	`vecextract`$(x,y,\{z\})$
transpose of vector or matrix x	`mattranspose`(x) or x`~`
adjoint of the matrix x	`matadjoint`(x)
eigenvectors/values of matrix x	`mateigen`(x)
characteristic/minimal polynomial of x	`charpoly`(x), `minpoly`
trace/determinant of matrix x	`trace`(x), `matdet`
Frobenius form of x	`matfrobenius`(x)
QR decomposition	`matqr`(x)

Constructors & Special Matrices

row vec. of *expr* eval'ed at $1 \le i \le n$	`vector`$(n,\{i\},\{expr\})$
col. vec. of *expr* eval'ed at $1 \le i \le n$	`vectorv`$(n,\{i\},\{expr\})$
matrix $1 \le i \le m$, $1 \le j \le n$	`matrix`$(m,n,\{i\},\{j\},\{expr\})$
define matrix by blocks	`matconcat`(B)
diagonal matrix with diagonal x	`matdiagonal`(x)
$n \times n$ identity matrix	`matid`(n)
Hessenberg form of square matrix x	`mathess`(x)
$n \times n$ Hilbert matrix $H_{ij} = (i+j-1)^{-1}$	`mathilbert`(n)
companion matrix to polynomial x	`matcompanion`(x)
Sylvester matrix of x	`polsylvestermatrix`(x)

Gaussian elimination

kernel of matrix x	`matker`$(x,\{flag\})$
intersection of column spaces of x and y	`matintersect`(x,y)
solve $M * X = B$ (M invertible)	`matsolve`(M,B)
as solve, modulo D (col. vector)	`matsolvemod`(M,D,B)
one sol of $M * X = B$	`matinverseimage`(M,B)
basis for image of matrix x	`matimage`(x)
supplement columns of x to get basis	`matsupplement`(x)
rows, cols to extract invertible matrix	`matindexrank`(x)
rank of the matrix x	`matrank`(x)

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	`mathnf`(x)
HNF of x where d is a multiple of $\det(x)$	`mathnfmod`(x,d)
elementary divisors of x	`matsnf`(x)
LLL-algorithm applied to columns of x	`qflll`$(x,\{flag\})$
like `qflll`, x is Gram matrix of lattice	`qflllgram`$(x,\{flag\})$
LLL-reduced basis for kernel of x	`matkerint`(x)
$\mathbf{Z}$-lattice $\longleftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz`(x,p)
signature of quad form ${}^t y * x * y$	`qfsign`(x)
decomp into squares of ${}^t y * x * y$	`qfgaussred`(x)
eigenvals/eigenvecs for real symmetric x	`qfjacobi`(x)
find up to m sols of ${}^t y * x * y \le b$	`qfminim`(x,b,m)
perfection rank of x	`qfperfection`(x)
v, $v[i]$:=number of sols of ${}^t y * x * y = i$	`qfrep`$(x,B,\{flag\})$
automorphism group of q	`qfauto`(q)
find isomorphism between q and Q	`qfisom`(q,Q)

Formal & p-adic Series

truncate power series or p-adic number	`truncate`(x)
valuation of x at p	`valuation`(x,p)

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	`taylor`(f,x)
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	`serconvol`(a,b)
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	`serlaplace`(f)
reverse power series F so $F(f(x)) = x$	`serreverse`(f)
Dirichlet series multiplication / division	`dirmul`, `dirdiv`(x,y)
Dirichlet Euler product (b terms)	`direuler`$(p=a,b,expr)$

Polynomials & Rational Functions

degree of f	`poldegree`(f)
coeff. of degree n of f, leading coeff.	`polcoeff`(f,n), `pollead`
gcd of coefficients of f	`content`(f)
replace x by y	`subst`(f,x,y)
evaluate f replacing vars by their value	`eval`(f)
replace polynomial expr. $T(x)$ by y in f	`substpol`(f,T,y)
replace $x_1,\dots,x_n$ by $y_1,\dots,y_n$ in f	`substvec`(f,x,y)
discriminant of polynomial f	`poldisc`(f)
resultant $R = \mathrm{Res}_v(f,g)$	`polresultant`$(f,g,\{v\})$
$[u,v,R]$, $xu+yv = \mathrm{Res}_v(f,g)$	`polresultantext`$(x,y,\{v\})$
derivative of f w.r.t. x	`deriv`$(f,\{x\})$
formal integral of f w.r.t. x	`intformal`$(f,\{x\})$
formal sum of f w.r.t. x	`sumformal`$(f,\{x\})$
reciprocal poly $x^{\deg f} f(1/x)$	`polrecip`(f)
interpol. pol. eval. at a	`polinterpolate`$(X,\{Y\},\{a\},\{$`&`$e\})$
initialize t for Thue equation solver	`thueinit`(f)
solve Thue equation $f(x,y) = a$	`thue`$(t,a,\{sol\})$

Roots and Factorization

number of real roots of f, $a < x \le b$	`polsturm`$(f,\{a\},\{b\})$
complex roots of f	`polroots`(f)
symmetric powers of roots of f up to n	`polsym`(f,n)
factor f	`factor`$(f,\{lim\})$
factor f mod p / roots	`factormod`(f,p), `polrootsmod`
factor f over $\mathbf{F}_{p^a}$ / roots	`factorff`(f,p,a), `polrootsff`
factor f over $\mathbf{Q}_p$ / roots	`factorpadic`(f,p,r), `polrootspadic`
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	`ffinit`$(p,n,\{x\})$
#{monic irred. $T \in \mathbf{F}_q[x], \deg T = n$}	`ffnbirred`(q,n)
p-adic root of f cong. to a mod p	`padicappr`(f,a)
Newton polygon of f for prime p	`newtonpoly`(f,p)
extensions of $\mathbf{Q}_p$ of degree N	`padicfields`(p,N)

Special Polynomials

n-th cyclotomic polynomial in var. v	`polcyclo`$(n,\{v\})$
d-th degree subfield of $\mathbf{Q}(\zeta_n)$	`polsubcyclo`$(n,d,\{v\})$
P_n, T_n/U_n, H_n	`pollegendre`, `polchebyshev`, `polhermite`

Transcendental and p-adic Functions

real, imaginary part of x	`real`(x), `imag`(x)
absolute value, argument of x	`abs`(x), `arg`(x)
square/nth root of x	`sqrt`(x), `sqrtn`$(x,n,\{$`&`$z\})$
trig functions	`sin, cos, tan, cotan`
inverse trig functions	`asin, acos, atan`
hyperbolic functions	`sinh, cosh, tanh`
inverse hyperbolic functions	`asinh, acosh, atanh`
exponential / natural log of x	`exp`, `log`
Euler Γ function, $\log\Gamma$, Γ'/Γ	`gamma`, `lngamma`, `psi`
incomplete gamma function ($y = \Gamma(s)$)	`incgam`$(s,x,\{y\})$
exponential integral $\int_x^\infty e^{-t}/t\,dt$	`eint1`(x)
error function $2/\sqrt{\pi}\int_x^\infty e^{-t^2}dt$	`erfc`(x)
dilogarithm of x	`dilog`(x)
m-th polylogarithm of x	`polylog`$(m,x,\{flag\})$
U-confluent hypergeometric function	`hyperu`(a,b,u)
Bessel $J_n(x)$, $J_{n+1/2}(x)$	`besselj`(n,x), `besseljh`(n,x)
Bessel I_ν, K_ν, H^1_ν, H^2_ν, N_ν	`(bessel)i`, `k`, `h1`, `h2`, `n`
Lambert W: x s.t. $xe^x = y$	`lambertw`(y)
Teichmuller character of p-adic x	`teichmuller`(x)

Elementary Arithmetic Functions

vector of binary digits of $\lvert x\rvert$	`binary`(x)
bit number n of integer x	`bittest`(x,n)
Hamming weight of integer x	`hammingweight`(x)
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round`$(x,\{$`&`$e\})$
truncate x	`truncate`$(x,\{$`&`$e\})$
gcd/LCM of x and y	`gcd`(x,y), `lcm`(x,y)
gcd of entries of a vector/matrix	`content`(x)

Primes and Factorization

add primes in v to prime table	`addprimes`(v)
Chebyshev $\pi(x)$, n-th prime p_n	`primepi`(x), `prime`(n)
vector of first n primes	`primes`(n)
smallest prime $\ge x$	`nextprime`(x)
largest prime $\le x$	`precprime`(x)
factorization of x	`factor`$(x,\{lim\})$
$n = df^2$, d squarefree/fundamental	`core`$(n,\{fl\})$, `coredisc`
recover x from its factorization	`factorback`$(f,\{e\})$

Divisors

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega`(n), `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors`(n), `numdiv`
sum of (k-th powers of) divisors of n	`sigma`$(n,\{k\})$

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	`binomial`(x,y)
Bernoulli number B_n as real/rational	`bernreal`(n), `bernfrac`
Bernoulli polynomial $B_n(x)$	`bernpol`$(n,\{x\})$
n-th Fibonacci number	`fibonacci`(n)
Stirling numbers $s(n,k)$ and $S(n,k)$	`stirling`$(n,k,\{flag\})$
number of partitions of n	`numbpart`(n)
Möbius μ-function	`moebius`(x)
Hilbert symbol of x and y (at p)	`hilbert`$(x,y,\{p\})$
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker`(x,y)
Dedekind sum $s(h,k)$	`sumdedekind`(h,k)

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi`(x)
multiplicative order of x (divides o)	`znorder`$(x,\{o\})$, `fforder`
primitive root mod q / `x.mod`	`znprimroot`(q), `ffprimroot`(x)
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar`(n)
discrete logarithm of x in base g	`znlog`$(x,g,\{o\})$, `fflog`

Miscellaneous

integer square / n-th root of x	`sqrtint`(x), `sqrtnint`(x,n)
solve $z \equiv x$ and $z \equiv y$	`chinese`(x,y)
minimal u,v so $xu+yv = \gcd(x,y)$	`gcdext`(x,y)
continued fraction of x	`contfrac`$(x,\{b\},\{lmax\})$
last convergent of continued fraction x	`contfracpnqn`(x)
rational approximation to x	`bestappr`(x,k), `bestapprPade`

True-False Tests

is x the disc. of a quadratic field?	`isfundamental`(x)
is x a prime?	`isprime`(x)
is x a strong pseudo-prime?	`ispseudoprime`(x)
is x square-free?	`issquarefree`(x)
is x a square?	`issquare`$(x,\{$`&`$n\})$
is x a perfect power?	`ispower`$(x,\{k\},\{$`&`$n\})$
is *pol* irreducible?	`polisirreducible`(pol)

Based on an earlier version by Joseph H. Silverman

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PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

Elliptic Curves

Elliptic curve initially given by 5-tuple $v = [a_1, a_2, a_3, a_4, a_6]$.

Initialize *ell* struct	`E = ellinit`$(v, \{Domain\})$

Points are `[x,y]`, the origin is `[0]`. Struct members accessed as `E.`*member*:

- All domains: `E.a1,a2,a3,a4,a6`, `b2,b4,b6,b8`, `c4,c6`, `disc`, `j`
- E defined over $\mathbf{R}$ or $\mathbf{C}$

x-coords. of points of order 2	`E.roots`
periods / quasi-periods	`E.omega,E.eta`
volume of complex lattice	`E.area`

- E defined over $\mathbf{Q}_p$

residual characteristic	`E.p`
If $\lvert j\rvert_p > 1$: Tate's $[u^2, u, q, [a, b]]$	`E.tate`

- E defined over $\mathbf{F}_q$

characteristic	`E.p`
$\#E(\mathbf{F}_q)$/cyclic structure/generators	`E.no`, `E.cyc`, `E.gen`

- E defined over $\mathbf{Q}$

generators of $E(\mathbf{Q})$ (require `elldata`)	`E.gen`
$[a_1, a_2, a_3, a_4, a_6]$ from j-invariant	`ellfromj`(j)
change curve E using $v = [u, r, s, t]$	`ellchangecurve`(E, v)
change point z using $v = [u, r, s, t]$	`ellchangepoint`(z, v)
add points $P + Q$ / $P - Q$	`elladd`(E, P, Q), `ellsub`
negate point	`ellneg`(E, P)
compute $n \cdot z$	`ellmul`(E, z, n)
n-division polynomial $f_n(x)$	`elldivpol`$(E, n, \{x\})$
check if z is on E	`ellisoncurve`(E, z)
order of torsion point z	`ellorder`(E, z)
y-coordinates of point(s) for x	`ellordinate`(E, x)
point $[\wp(z), \wp'(z)]$ corresp. to z	`elleztopoint`(E, z)
complex z such that $p = [\wp(z), \wp'(z)]$	`ellpointtoz`(E, p)

Curves over finite fields, Pairings

random point on E	`random`(E)
$\#E(\mathbf{F}_q)$	`ellcard`(E)
structure $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$ of $E(\mathbf{F}_q)$	`ellgroup`(E)
Weil pairing of m-torsion pts x, y	`ellweilpairing`(E, x, y, m)
Tate pairing of x, y; x m-torsion	`elltatepairing`(E, x, y, m)
Discrete log, find n s.t. $P = [n]Q$	`elllog`$(E, P, Q, \{ord\})$

Curves over Q and the L-function

canonical bilinear form taken at z_1, z_2	`ellbil`(E, z_1, z_2)
canonical height of z	`ellheight`$(E, z, \{flag\})$
height regulator matrix for pts in x	`ellheightmatrix`(E, x)
cond, min mod, Tamagawa num `[`N, v, c`]`	`ellglobalred`(E)
reduction of $y^2 + Qy = P$ (genus 2)	`genus2red`$(Q, P, \{p\})$
Kodaira type of p-fiber of E	`elllocalred`(E, p)
minimal model of $E/\mathbf{Q}$	`ellminimalmodel`$(E, \{\&v\})$
p-th coeff a_p of L-function, p prime	`ellap`(E, p)
k-th coeff a_k of L-function	`ellak`(E, k)
vector of first n a_k's in L-function	`ellan`(E, n)
$L(E, s)$	`elllseries`(E, s)
$L^{(r)}(E, 1)$	`ellL1`(E, r)
return a Heegner point on E of rank 1	`ellheegner`(E)
order of vanishing at 1	`ellanalyticrank`$(E, \{eps\})$
root number for $L(E, .)$ at p	`ellrootno`$(E, \{p\})$
torsion subgroup with generators	`elltors`(E)
modular parametrization of E	`elltaniyama`(E)

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Elldata package, Cremona's database:

db code $\leftrightarrow$ $[conductor, class, index]$	`ellconvertname`(s)
generators of Mordell-Weil group	`ellgenerators`(E)
look up E in database	`ellidentify`(E)
all curves matching criterion	`ellsearch`(N)
loop over curves with cond. from a to b	`forell`(E, a, b, seq)

Elliptic & Modular Functions

$w = [\omega_1, \omega_2]$ or *ell* struct (`E.omega`), $\tau = \omega_1/\omega_2$.

arithmetic-geometric mean	`agm`(x, y)
elliptic j-function $1/q + 744 + \cdots$	`ellj`(x)
Weierstrass $\sigma/\wp/\zeta$ function	`ellsigma`(w, z), `ellwp`, `ellzeta`
periods/quasi-periods	`ellperiods`$(E, \{flag\})$, `elleta`(w)
$(2i\pi/\omega_2)^k E_k(\tau)$	`elleisnum`$(w, k, \{flag\})$
modified Dedekind η func. $\prod(1 - q^n)$	`eta`$(x, \{flag\})$
Jacobi sine theta function	`theta`(q, z)
k-th derivative at z=0 of `theta`(q, z)	`thetanullk`(q, k)
Weber's f functions	`weber`$(x, \{flag\})$
Riemann's zeta $\zeta(s) = \sum n^{-s}$	`zeta`(s)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d)	`Qfb`$(a, b, c, \{d\})$
reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$)	`qfbred`$(x, \{flag\}, \{D\}, \{l\}, \{s\})$
composition of forms	x`*`y or `qfbnucomp`(x, y, l)
n-th power of form	x`^`n or `qfbnupow`(x, n)
composition without reduction	`qfbcompraw`(x, y)
n-th power without reduction	`qfbpowraw`(x, n)
prime form of disc. x above prime p	`qfbprimeform`(x, p)
class number of disc. x	`qfbclassno`(x)
Hurwitz class number of disc. x	`qfbhclassno`(x)
Solve $Q(x, y) = p$ in integers, p prime	`qfbsolve`(Q, p)

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$	`quadgen`(x)
minimal polynomial of ω	`quadpoly`(x)
discriminant of $\mathbf{Q}(\sqrt{D})$	`quaddisc`(x)
regulator of real quadratic field	`quadregulator`(x)
fundamental unit in real $\mathbf{Q}(x)$	`quadunit`(x)
class group of $\mathbf{Q}(\sqrt{D})$	`quadclassunit`$(D, \{flag\}, \{t\})$
Hilbert class field of $\mathbf{Q}(\sqrt{D})$	`quadhilbert`$(D, \{flag\})$
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$	`quadray`$(D, f, \{flag\})$

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure *nf*	`nfinit`$(f, \{flag\})$

nf members:

polynomial defining *nf*, $f(\theta) = 0$	*nf*`.pol`
number of real/complex places	*nf*`.r1/r2/sign`
discriminant of *nf*	*nf*`.disc`
T_2 matrix	*nf*`.t2`
vector of roots of f	*nf*`.roots`
integral basis of $\mathbf{Z}_K$ as powers of θ	*nf*`.zk`
different	*nf*`.diff`
codifferent	*nf*`.codiff`
index	*nf*`.index`

recompute *nf* using current precision	`nfnewprec`(nf)
init relative *rnf* given by $g = 0$ over K	`rnfinit`(nf, g)
init*bnf* structure	`bnfinit`$(f, \{flag\})$

bnf members: same as *nf*, plus

underlying *nf*	*bnf*`.nf`
classgroup	*bnf*`.clgp`
regulator	*bnf*`.reg`
fundamental units	*bnf*`.fu`
torsion units	*bnf*`.tu`

compute a *bnf* from small *bnf*	`bnfinit`$(sbnf)$
add S-class group and units, yield *bnf* s	`bnfsunit`(nf, S)
init class field structure *bnr*	`bnrinit`$(bnf, m, \{flag\})$

bnr members: same as *bnf*, plus

underlying *bnf*	*bnr*`.bnf`
big ideal structure	*bnr*`.bid`
modulus	*bnr*`.mod`
structure of $(\mathbf{Z}_K/m)^*$	*bnr*`.zkst`

Basic Number Field Arithmetic (nf)

Elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis *nf*`.zk`). Basic operations (prefix `nfelt`): (`nfelt`)`add`, `mul`, `pow`, `div`, `diveuc`, `mod`, `divrem`, `val`, `trace`, `norm`

express x on integer basis	`nfalgtobasis`(nf, x)
express element x as a polmod	`nfbasistoalg`(nf, x)
reverse polmod $a = A(X) \bmod T(X)$	`modreverse`(a)
integral basis of field def. by $f = 0$	`nfbasis`(f)
field discriminant of field $f = 0$	`nfdisc`(f)
smallest poly defining $f = 0$ (slow)	`polredabs`$(f, \{flag\})$
small poly defining $f = 0$ (fast)	`polredbest`$(f, \{flag\})$
are fields $f = 0$ and $g = 0$ isomorphic?	`nfisisom`(f, g)
is field $f = 0$ a subfield of $g = 0$?	`nfisincl`(f, g)
compositum of $f = 0$, $g = 0$	`polcompositum`$(f, g, \{flag\})$
subfields (of degree d) of *nf*	`nfsubfields`$(nf, \{d\})$
roots of unity in *nf*	`nfrootsof1`(nf)
roots of g belonging to *nf*	`nfroots`$(\{nf\}, g)$
factor g in *nf*	`nffactor`(nf, g)
factor g mod prime pr in *nf*	`nffactormod`(nf, g, pr)
conjugates of a root θ of *nf*	`nfgaloisconj`$(nf, \{flag\})$
apply Galois automorphism s to x	`nfgaloisapply`(nf, s, x)
quadratic Hilbert symbol (at p)	`nfhilbert`$(nf, a, b, \{p\})$

Linear and algebraic relations

poly of degree $\le k$ with root $x \in \mathbf{C}$	`algdep`(x, k)
alg. dep. with pol. coeffs for series s	`seralgdep`(s, x, y)
small linear rel. on coords of vector x	`lindep`(x)

Dedekind Zeta Function ζ_K, Hecke L series

ζ_K as Dirichlet series, $N(I) < b$	`dirzetak`(nf, b)
init *nfz* for field $f = 0$	`zetakinit`(f)
compute $\zeta_K(s)$	`zetak`$(nfz, s, \{flag\})$
Artin root number of K	`bnrrootnumber`$(bnr, chi, \{flag\})$
$L(1, \chi)$, for all χ trivial on H	`bnrL1`$(bnr, \{H\}, \{flag\})$

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually $bnr, subgp$ or $bnf, module, \{subgp\}$

remove GRH assumption from *bnf*	`bnfcertify`(bnf)
expo. of ideal x on class gp	`bnfisprincipal`$(bnf, x, \{flag\})$
expo. of ideal x on ray class gp	`bnrisprincipal`$(bnr, x, \{flag\})$
expo. of x on fund. units	`bnfisunit`(bnf, x)
as above for S-units	`bnfissunit`$(bnfs, x)$
signs of real embeddings of *bnf*`.fu`	`bnfsignunit`(bnf)
narrow class group	`bnfnarrow`(bnf)

PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

Class Field Theory

ray class number for mod. m `bnrclassno`(bnf, m)
discriminant of class field ext `bnrdisc`$(a_1, \{a_2\}, \{a_3\})$
ray class numbers, l list of mods `bnrclassnolist`(bnf, l)
discriminants of class fields `bnrdisclist`$(bnf, l, \{arch\}, \{flag\})$
decode output from `bnrdisclist` `bnfdecodemodule`(nf, fa)
is modulus the conductor? `bnrisconductor`$(a_1, \{a_2\}, \{a_3\})$
conductor of character chi `bnrconductorofchar`(bnr, chi)
conductor of extension `bnrconductor`$(a_1, \{a_2\}, \{a_3\}, \{flag\})$
conductor of extension def. by g `rnfconductor`(bnf, g)
Artin group of ext. def'd by g `rnfnormgroup`(bnr, g)
subgroups of bnr, index $<= b$ `subgrouplist`$(bnr, b, \{flag\})$
rel. eq. for class field def'd by sub `rnfkummer`$(bnr, sub, \{d\})$
same, using Stark units (real field) `bnrstark`$(bnr, sub, \{flag\})$

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ? `nfisideal`(nf, id)
is x principal in bnf ? `bnfisprincipal`(bnf, x)
give `[`a, b`]`, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ `idealtwoelt`$(nf, x, \{a\})$
put ideal a $(a\mathbf{Z}_K + b\mathbf{Z}_K)$ in HNF form `idealhnf`$(nf, a, \{b\})$
norm of ideal x `idealnorm`(nf, x)
minimum of ideal x (direction v) `idealmin`(nf, x, v)
LLL-reduce the ideal x (direction v) `idealred`$(nf, x, \{v\})$

Ideal Operations

add ideals x and y `idealadd`(nf, x, y)
multiply ideals x and y `idealmul`$(nf, x, y, \{flag\})$
intersection of ideals x and y `idealintersect`$(nf, x, y, \{flag\})$
n-th power of ideal x `idealpow`$(nf, x, n, \{flag\})$
inverse of ideal x `idealinv`(nf, x)
divide ideal x by y `idealdiv`$(nf, x, y, \{flag\})$
Find $(a, b) \in x \times y$, $a + b = 1$ `idealaddtoone`$(nf, x, \{y\})$
coprime integral A, B such that $x = A/B$ `idealnumden`(nf, x)

Primes and Multiplicative Structure

factor ideal x in nf `idealfactor`(nf, x)
expand ideal factorization in nf `idealfactorback`(nf, f, e)
decomposition of prime p in nf `idealprimedec`(nf, p)
valuation of x at prime ideal pr `idealval`(nf, x, pr)
weak approximation theorem in nf `idealchinese`(nf, x, y)
give bid =structure of $(\mathbf{Z}_K/id)^*$ `idealstar`$(nf, id, \{flag\})$
discrete log of x in $(\mathbf{Z}_K/bid)^*$ `ideallog`(nf, x, bid)
`idealstar` of all ideals of norm $\leq b$ `ideallist`$(nf, b, \{flag\})$
add Archimedean places `ideallistarch`$(nf, b, \{ar\}, \{flag\})$
init `prmod` structure `nfmodprinit`(nf, pr)
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ `nfkermodpr`$(nf, M, prmod)$
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ `nfsolvemodpr`$(nf, M, B, prmod)$

Galois theory over Q

Galois group of field $\mathbf{Q}[x]/(f)$ `polgalois`(f)
initializes a Galois group structure G `galoisinit`$(pol, \{den\})$
action of p in nfgaloisconj form `galoispermtopol`$(G, \{p\})$
identify as abstract group `galoisidentify`(G)
export a group for GAP/MAGMA `galoisexport`$(G, \{flag\})$
subgroups of the Galois group G `galoissubgroups`(G)
is subgroup H normal? `galoisisnormal`(G, H)
subfields from subgroups `galoissubfields`$(G, \{flag\}, \{v\})$
fixed field `galoisfixedfield`$(G, perm, \{flag\}, \{v\})$
Frobenius at maximal ideal P `idealfrobenius`(nf, G, P)
ramification groups at P `idealramgroups`(nf, G, P)
is G abelian? `galoisisabelian`$(G, \{flag\})$
abelian number fields/$\mathbf{Q}$ `galoissubcyclo(N,H,{`$flag$`},{v})`
query the `galpol` package `galoisgetpol(a,b,{s})`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.
absolute equation of L `rnfequation`$(nf, T, \{flag\})$
is L/K abelian? `rnfisabelian`(nf, T)
relative `nfalgtobasis` `rnfalgtobasis`(rnf, x)
relative `nfbasistoalg` `rnfbasistoalg`(rnf, x)
relative `idealhnf` `rnfidealhnf`(rnf, x)
relative `idealmul` `rnfidealmul`(rnf, x, y)
relative `idealtwoelt` `rnfidealtwoelt`(rnf, x)

Lifts and Push-downs

absolute $\to$ relative repres. for x `rnfeltabstorel`(rnf, x)
relative $\to$ absolute repres. for x `rnfeltreltoabs`(rnf, x)
lift x to the relative field `rnfeltup`(rnf, x)
push x down to the base field `rnfeltdown`(rnf, x)
idem for x ideal: `(rnfideal)reltoabs`, `abstorel`, `up`, `down`

Norms

absolute norm of ideal x `rnfidealnormabs`(rnf, x)
relative norm of ideal x `rnfidealnormrel`(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ `bnfisintnorm`(bnf, x)
is $x \in \mathbf{Q}$ a norm from K? `bnfisnorm`$(bnf, x, \{flag\})$
initialize T for norm eq. solver `rnfisnorminit`$(K, pol, \{flag\})$
is $a \in K$ a norm from L? `rnfisnorm`$(T, a, \{flag\})$

Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$-module

relative `polred` `rnfpolred`(nf, T)
relative `polredabs` `rnfpolredabs`(nf, T)
characteristic poly. of a mod T `rnfcharpoly`$(nf, T, a, \{v\})$
relative Dedekind criterion, prime pr `rnfdedekind`(nf, T, pr)
discriminant of relative extension `rnfdisc`(nf, T)
pseudo-basis of $\mathbf{Z}_L$ `rnfpseudobasis`(nf, T)

General $\mathbf{Z}_K$-modules: $M = [\text{matrix}, \text{vec. of ideals}] \subset L$

relative HNF / SNF `nfhnf`(nf, M), `nfsnf`
reduced basis for M `rnflllgram`(nf, T, M)
determinant of pseudo-matrix M `rnfdet`(nf, M)
Steinitz class of M `rnfsteinitz`(nf, M)
$\mathbf{Z}_K$-basis of M if $\mathbf{Z}_K$-free, or 0 `rnfhnfbasis`(bnf, M)
n-basis of M, or $(n+1)$-generating set `rnfbasis`(bnf, M)
is M a free $\mathbf{Z}_K$-module? `rnfisfree`(bnf, M)

Graphic Functions

crude graph of $expr$ between a and b `plot`$(X = a, b, expr)$

High-resolution plot (immediate plot)

plot $expr$ between a and b `ploth`$(X = a, b, expr, \{flag\}, \{n\})$
plot points given by lists lx, ly `plothraw`$(lx, ly, \{flag\})$
terminal dimensions `plothsizes`()

Rectwindow functions

init window w, with size x,y `plotinit`(w, x, y)
erase window w `plotkill`(w)
copy w to w_2 with offset (dx, dy) `plotcopy`(w, w_2, dx, dy)
clips contents of w `plotclip`(w)
scale coordinates in w `plotscale`(w, x_1, x_2, y_1, y_2)
`ploth` in w `plotrecth`$(w, X = a, b, expr, \{flag\}, \{n\})$
`plothraw` in w `plotrecthraw`$(w, data, \{flag\})$
draw window w_1 at (x_1, y_1), ... `plotdraw`$([[w_1, x_1, y_1], \ldots])$

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor`(w, c)
current position of cursor in w `plotcursor`(w)
write s at cursor's position `plotstring`(w, s)
move cursor to (x, y) `plotmove`(w, x, y)
move cursor to $(x + dx, y + dy)$ `plotrmove`(w, dx, dy)
draw a box to (x_2, y_2) `plotbox`(w, x_2, y_2)
draw a box to $(x + dx, y + dy)$ `plotrbox`(w, dx, dy)
draw polygon `plotlines`$(w, lx, ly, \{flag\})$
draw points `plotpoints`(w, lx, ly)
draw line to $(x + dx, y + dy)$ `plotrline`(w, dx, dy)
draw point $(x + dx, y + dy)$ `plotrpoint`(w, dx, dy)
draw point $(x + dx, y + dy)$ `plotrpoint`(w, dx, dy)

Postscript Functions

as `ploth` `psploth`$(X = a, b, expr, \{flag\}, \{n\})$
as `plothraw` `psplothraw`$(lx, ly, \{flag\})$
as `plotdraw` `psdraw`$([[w_1, x_1, y_1], \ldots])$

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