Viskovatov's algorithm, all four variables: $f_1^{[k]}$, $f_0^{[k]}$, $c_1^{[k]}$, $c_0^{[k]}$.

$$f_1 = \exp(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5)$$
$$f_0 = 1 + O(z^1)$$

Zero coefficient:

$$c_1 = 1 \quad (\text{from } f_1)$$
$$c_0 = 1 \quad (\text{from } f_0)$$
$$c_1^{[k]} = \text{zero coefficient from } f_1^{[k]},\ k \geq 1$$
$$c_0^{[k]} = \text{zero coefficient from } f_0^{[k]},\ k \geq 1$$

First coefficient:

$$f_1^{[1]} = f_0 = 1 + O(z^1)$$
$$f_0^{[1]} = \frac{1}{z}\left(f_1 - \frac{c_1}{c_0} f_0\right) = \frac{1}{z}\left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5) - \frac{1}{1}\left(1 + O(z^1)\right)\right) =$$
$$= 1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)$$
$$c_1^{[1]} = 1$$
$$c_0^{[1]} = 1$$

Second coefficient:

$$f_1^{[2]} = f_0^{[1]} = 1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)$$
$$f_0^{[2]} = \frac{1}{z}\left(f_1^{[1]} - \frac{c_1^{[1]}}{c_0^{[1]}} f_0^{[1]}\right) = \frac{1}{z}\left(1 + O(z^1) - \frac{1}{1}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)\right)\right) =$$
$$= \frac{1}{z}\left(1 - 1 - \frac{z}{2} - \frac{z^2}{6} - \frac{z^3}{24} + O(z^4)\right) =$$
$$= -\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)$$
$$c_1^{[2]} = 1$$
$$c_0^{[2]} = -\frac{1}{2}$$

Third coefficient:

$$
\begin{aligned}
f_1^{[3]} &= f_0^{[2]} = -\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3) \\
f_0^{[3]} &= \frac{1}{z}\left(f_1^{[2]} - \frac{c_1^{[2]}}{c_0^{[2]}} f_0^{[2]}\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - \frac{1}{-1/2}\left(-\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)\right)\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - 1 - \frac{z}{3} - \frac{z^2}{12} + O(z^3)\right) \\
&= \frac{1}{6} + \frac{z}{12} + O(z^2) \\
c_1^{[3]} &= -\frac{1}{2} \\
c_0^{[3]} &= \frac{1}{6}
\end{aligned}
$$

The coefficients for the continued fraction are $c_1^{[k]}/c_0^{[k]}$, $k = 1, 2, 3, \ldots$

$$
\begin{aligned}
\text{coeff}_0 &= \frac{c_1}{c_0} = 1 \\
\text{coeff}_1 &= \frac{c_1^{[1]}}{c_0^{[1]}} = 1 \\
\text{coeff}_2 &= \frac{c_1^{[2]}}{c_0^{[2]}} = \frac{1}{-1/2} = -2 \\
\text{coeff}_3 &= \frac{c_1^{[3]}}{c_0^{[3]}} = \frac{-1/2}{1/6} = -3
\end{aligned}
$$

By computer program, the first eight are:

$$
\begin{aligned}
\overrightarrow{c_1} &= [1, 1, -1/2, 1/6, 1/12, 1/60, -1/120, 1/840] \\
\overrightarrow{c_0} &= [1, -1/2, 1/6, 1/12, 1/60, -1/120, 1/840, 1/1680] \\
\overrightarrow{c_1/c_0} &= [1, -2, -3, 2, 5, -2, -7, 2]
\end{aligned}
$$

Continued fraction is:

$$
\frac{f_1}{f_0} = 1 + \frac{z}{1} + \frac{z}{-2} + \frac{z}{-3} + \frac{z}{2} + \frac{z}{5} + \frac{z}{-2} + \frac{z}{-7} + \frac{z}{2}.
$$

Padé approximant is:

$$
[4/4] = \frac{z^4 + 20z^3 + 180z^2 + 840z + 1680}{z^4 - 20z^3 + 180z^2 - 840z + 1680}.
$$

First optimization:

$$f_0^{[k]} = \frac{1}{z}\left(f_0^{[k-2]} - \frac{c_1^{[k-1]}}{c_0^{[k-1]}} f_0^{[k-1]}\right),\ k \geq 3$$
$$c_1^{[k]} = \text{zero coefficient from } f_0^{[k-1]},\ k \geq 2$$
$$c_0^{[k]} = \text{zero coefficient from } f_0^{[k]},\ k \geq 1$$

Viskovatov's algorithm, no $f_1^{[k]}$, only $f_0^{[k]}$, $c_1^{[k]}$, $c_0^{[k]}$.

$$f_1 = \exp(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5)$$
$$f_0 = 1 + O(z^1)$$

Zero coefficient:

$$c_1 = 1$$
$$c_0 = 1$$

First coefficient:

$$f_0^{[1]} = \frac{1}{z}\left(f_1 - \frac{c_1}{c_0} f_0\right) = \frac{1}{z}\left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5) - \frac{1}{1}\left(1 + O(z^1)\right)\right) =$$
$$= 1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)$$
$$c_1^{[1]} = 1 \quad (\text{equals } c_0)$$
$$c_0^{[1]} = 1 \quad \left(\text{from } f_0^{[1]}\right)$$

Second coefficient:

$$f_0^{[2]} = \frac{1}{z}\left(f_0 - \frac{c_1^{[1]}}{c_0^{[1]}} f_0^{[1]}\right) = \frac{1}{z}\left(1 + O(z^1) - \frac{1}{1}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)\right)\right) =$$
$$= \frac{1}{z}\left(1 - 1 - \frac{z}{2} - \frac{z^2}{6} - \frac{z^3}{24} + O(z^4)\right) =$$
$$= -\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)$$
$$c_1^{[2]} = 1 \quad \left(\text{from } f_0^{[1]}\right)$$
$$c_0^{[2]} = -\frac{1}{2} \quad \left(\text{from } f_0^{[2]}\right)$$

Third coefficient:

$$
\begin{aligned}
f_0^{[3]} &= \frac{1}{z}\left(f_0^{[1]} - \frac{c_1^{[2]}}{c_0^{[2]}} f_0^{[2]}\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - \frac{1}{-1/2}\left(-\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)\right)\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - 1 - \frac{z}{3} - \frac{z^2}{12} + O(z^3)\right) \\
&= \frac{1}{6} + \frac{z}{12} + O(z^2) \\
c_1^{[3]} &= -\frac{1}{2} \quad \left(\text{from } f_0^{[2]}\right) \\
c_0^{[3]} &= \frac{1}{6} \quad \left(\text{from } f_0^{[3]}\right)
\end{aligned}
$$

The coefficients for the continued fraction are the same as before.

Second optimization:

$$f_0^{[k]} = \frac{1}{z}\left(f_0^{[k-2]} - \frac{c_1^{[k-1]}}{c_0^{[k-1]}} f_0^{[k-1]}\right),\ k \geq 3$$
$$c^{[k+1]} = \text{zero coefficient from } f_0^{[k]},\ k \geq 1$$

Viskovatov's algorithm, no $f_1^{[k]}$, $c_1^{[k]}$, only $f_0^{[k]}$, $c_0^{[k]}$.

$$f_1 = \exp(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5)$$
$$f_0 = 1 + O(z^1)$$

Zero coefficient:

$$c_1 = 1$$
$$c_0 = 1$$
$$c^{[1]} = c_0 = 1$$

First coefficient:

$$f_0^{[1]} = \frac{1}{z}\left(f_1 - \frac{c_1}{c_0} f_0\right) = \frac{1}{z}\left(1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + O(z^5) - \frac{1}{1}\left(1 + O(z^1)\right)\right) =$$
$$= 1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)$$
$$c^{[2]} = 1$$

Second coefficient:

$$f_0^{[2]} = \frac{1}{z}\left(f_0 - \frac{c_1^{[1]}}{c_0^{[1]}} f_0^{[1]}\right) = \frac{1}{z}\left(1 + O(z^1) - \frac{1}{1}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4)\right)\right) =$$
$$= \frac{1}{z}\left(1 - 1 - \frac{z}{2} - \frac{z^2}{6} - \frac{z^3}{24} + O(z^4)\right) =$$
$$= -\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)$$
$$c^{[3]} = -\frac{1}{2}$$

Third coefficient:

$$
\begin{aligned}
f_0^{[3]} &= \frac{1}{z}\left(f_0^{[1]} - \frac{c_1^{[2]}}{c_0^{[2]}} f_0^{[2]}\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - \frac{1}{-1/2}\left(-\frac{1}{2} - \frac{z}{6} - \frac{z^2}{24} + O(z^3)\right)\right) = \\
&= \frac{1}{z}\left(1 + \frac{z}{2} + \frac{z^2}{6} + \frac{z^3}{24} + O(z^4) - 1 - \frac{z}{3} - \frac{z^2}{12} + O(z^3)\right) \\
&= \frac{1}{6} + \frac{z}{12} + O(z^2) \\
c^{[4]} &= \frac{1}{6}
\end{aligned}
$$

The first eight are:

$$\overrightarrow{c} = [1, 1, -1/2, 1/6, 1/12, 1/60, -1/120, 1/840, 1/1680]$$

Compare with $\overrightarrow{c_1}$ and $\overrightarrow{c_0}$ from before. The length of $\overrightarrow{c}$ is 9, since we compute from $f_0^{[1]}$ to $f_0^{[8]}$ for all 8 coefficients for the continued fraction. Computing the coefficients for the continued fraction by:

$$
\begin{aligned}
\text{coeff}_0 &= \frac{c_1}{c_0} = 1 \\
\text{coeff}_1 &= \frac{c^{[1]}}{c^{[2]}} = 1 \\
\text{coeff}_2 &= \frac{c^{[2]}}{c^{[3]}} = \frac{1}{-1/2} = -2 \\
\text{coeff}_3 &= \frac{c^{[3]}}{c^{[4]}} = \frac{-1/2}{1/6} = -3 \\
\text{coeff}_j &= \frac{c^{[j]}}{c^{[j+1]}},\ k = 1, 2, 3, \ldots
\end{aligned}
$$

This optimization might be useful for computer programs, since it reduces the amount of variables computed.