# Readme for HEPA Computation 1.0.0

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This program computes a rational approximant to a function, given by its Taylor expansion at the origin. Then the zeros of the polynomials of the rational approximant are plotted on the complex plane. This is achieved by using two computer programs, `PARI/GP` and `gnuplot`.

## 1 Installing required software

This program, `HEPAComp.jar`, is written in the Java programming language, and requires Java Virtual Machine to run. Please visit `http://java.com` and install `Java`.

`PARI/GP` and `gnuplot` are supplied as Windows executables in the subfolders of the archive as `.\pari\gp64-2-7-5.exe` and `.\gnuplot\gnuplot.exe`.

There is also a PDF viewer, `SumatraPDF`, that is `.\sumatra\SumatraPDF.exe`. The PDF viewer can be changed from `config.ini`.

The program is ready to go on Windows, simply double-click on `HEPAComp.jar` and the user interface should appear. Press Start, wait a while, then press Create plot. The result should be displayed in a window.

## 2 Installing required software on Linux

Please use the distribution's package manager to install `Java` and `gnuplot`.

`PARI/GP` should be compiled manually, it takes 5 minutes and the result is a single executable. The recommended `PARI/GP` version is supplied as `./pari/pari-2.7.5.tar.gz`.

Required packages: `gcc`, `libc6-dev`, `libgmp-dev`, which can be installed by this command on Ubuntu based distributions:

```
sudo apt-get install gcc libc6-dev libgmp-dev
```

Please use the following commands to compile `PARI/GP`:

```
tar -xvf pari-2.7.5.tar.gz
cd ./pari-2.7.5
./Configure
make bench
cp ./Olinux-x86_64/gp-sta ../gp
```

Start the HEPAComp program by executing `HEPAComp.sh` or by using the command: `java -jar HEPAComp.jar`.

Once in the program, press Start, then press Create plot. The result should be displayed in a window. If not, please edit `config.ini` with the appropriate program paths.

## 3 Computational options for PARI/GP

This program can compute the following approximants with the help of `PARI/GP`:

- Padé approximant of a function $F(z)$,
- Hermite-Padé approximant of $[1, F, G]$,
- Hermite-Padé approximant of $[1, F, G, H]$,
- Two-point Padé approximant of $F$ and $G$,
- Discriminant for Hermite-Padé approximant of $[1, F, G]$,
- Special index for Hermite-Padé approximant of $[1, F, G]$.

Press Start once and wait until it completes, that is the text "Competed in" appears. Then press Create plot. Select different options for the plot and then press Create plot again. If the computation does not start, please edit `config.ini` and change the version of `PARI/GP`.

All files are placed in the subdirectory `output`. Open the log file to see everything about the current session.

Session name: Determined automatically by the type of the approximant, the degree of the polynomials and four random digits. Select the checkbox to input the name. The name can contain digits 0-9, letters A-z, a-z, underscore, dash and round brackets ().

The session name is checked against all `*.log` files in the `output` directory The name should be unique, otherwise the data files may contain several sets of points (since `PARI/GP` appends the information in the files).

Precision (digits): Significant decimal digits after the decimal point. This must be an integer number, at least 100. By default this is set to 1000. Incresing this number increases somewhat the memory used and increases a lot the computational time.

Polynomial degree: "Diagonal" approximation: All polynomials (which are the result of the approximation) have this degree. Denote the degree by $N$. Only the first polynomial, named $Q_0(z)$, may have $N - 1$ or $N$ zeros. The rest of the polynomials are guaranteed to have exactly $N$ zeros. The degree must be an integer value, at least 1. By default this is set to 30. Increasing this number significantly increases the memory and computation time.

"Non-diagonal" approximation: input several integer values separated by comma. Their count must be equal to the approximant's number of polynomials. PADE, TPPA, TPPB have by 2 polynomials, HEPA, HEPAD, HEPAS – 3, MHEPA – 4.

Multiple approximations: Select the checkbox to set an integer number, greater than the initial degree, to compute sevaral approximations. This only works with "diagonal" apprximation. Number of 50 means that appoximants for $30, 31, 32, \ldots 50$ are computed. By default the checkbox is not selected.

Taylor expansion: Maximum order of the Taylor expansion. The formula is: add the degrees of the polynomials, the number of the polynomials and subtract 1. For "diagonal"

approximation the formula is: $pol * N + pol - 1$. For example, let $N = 30$. PADE has 2 polynomials, then the maximal order is 61. HEPA has 3 polynomials, order is 92. MHEPA has 4 polynomials, order is 123.

Input this number only for the error "polcoeff: non existent component in truecoeff". This happens when the function is divided by $z$, $z^2$, $z^3$. Increase the number by 1, 2, 3, respectively.

Approximant type: Select between the following:

- (PADE) Padé approximant of $F(z)$,
- (HEPA) Hermite-Padé approximant of $[1, F, G]$,
- (MHEPA) Hermite-Padé approximant of $[1, F, G, H]$,
- (TPPA) Two-point Padé approximant of $F$, $G$ with $/N + 1, N/$,
- (TPPB) Two-point Padé approximant of $F$, $G$ with $/N + 1, N/$,
- (HEPAD) Discriminant for Hermite-Padé approximant of $[1, F, G]$,
- (HEPAS) Special index for Hermite-Padé approximant of $[1, F, G]$.

The functions: Three functions can be typed in or selected by the dropdown menu. Their internal names are funF, funG and funH. Note that funF is evaluated first, then funG and finally funH.

`PARI/GP` uses real numbers if there is at least one real number in the function. Use capital letter I for the imaginary unit. Use `^(1/2)` for square root and `^(1/3)` for cube root. Use exp, log, sin, cos, tan, cotan for the transcendental functions. Reference the previous functions by funF and funG, like `funF^2`, `funG^(-2/3)`, etc.

After calculations: The approximant is calculated at the origin. Select between staying at the origin and sending to the infinity point.

Transformations: These transformations are applied on every zero of every polynomial. Choose between Joukovsky transform and inverse Joukovsky transform.

Memory (in megabytes): Increase this for the error "the PARI stack overflows". By default this is set to 1200.

Start: When the button is pressed, all data from the panel is collected and a script named `hepa30_1000_8329.gp` is created. Then `PARI/GP` reads this script, makes the computations, finds the zeros of the polynomials, and saves them as `hepa30_1000_8329q0.gpd`, `hepa30_1000_8329q1.gpd` and `hepa30_1000_8329q2.gpd` for $Q_0(z)$, $Q_1(z)$, and $Q_2(z)$ respectively. These files have to be parsed for `gnuplot`, therefore the final data files are saved as `*.gpt`.

Please do not press Start again when a computation is running. There might be errors. Please one by one.

Stop: If a computation is in progress, press this button first before exiting the program. Otherwise the process "gp.exe" continues running in the background. If that happens, use the operating system's process manager to terminate it.

## 4 Plotting options for gnuplot

List of sessions: The computed approximants appear in this list. Existing sessions can be added with the "plus" button. Sessions can be rearranged with the arrow buttons. Remove sessions by the fourth button.

Multiple sessions can be added or deleted at once. Hold the Ctrl key to select several sessions, hold the Shift key to select a range of sessions. This applies both to the file chooser and the list of sessions.

Select a session with the mouse by left-clicking on it. Then press the Polynomials button, a small window should appear, it allows choosing which polynomials to plot. The polynomials are referenced by 0,1,2,3 in the session name, and by Q0,Q1,Q2,Q3 in the window.

Polynomials can also be chosen by double-clicking with the mouse on the session name, or by right-clicking on a session name, and using the menu that appears.

The first session on the list is plotted first, then the next session is plotted on top of it, and so on. The last session is plotted on top of all other sessions. Rearrange the sessions with the arrow buttons.

Range of the plot: The range for the X and Y axes can be automatically determined by `gnuplot` or manually entered. Some common ranges are there for convenience.

Range for X and Y axis: The values can be integer and real numbers.

File type: Choose between Portable Document Format and Encapsulated PostScript. The `gnuplot` backend is "cairo graphics" and the respective terminals are pdfcairo and epscairo.

Create plot: This button creates a plot from all available sessions. The `gnuplot` script file is named `result_20160320_160325.plt`. This file contains the sessions, which have chosen polynomials. All polynomials are plotted by default, choose polynomials with the Polynomials button.

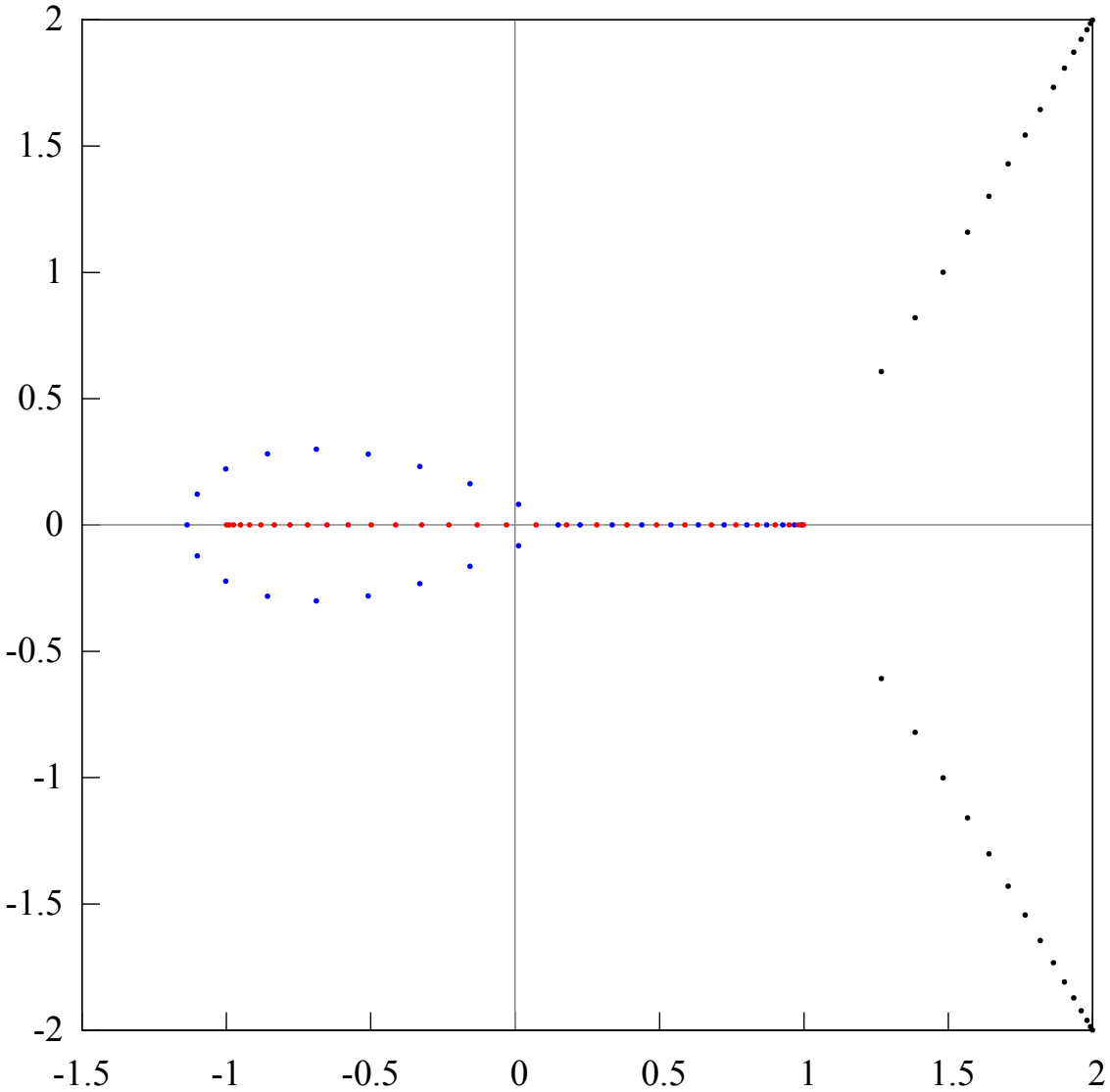
Then `gnuplot` creates the plot from the script as `result_20160320_160325.pdf`. Finally, the file viewer opens the PDF file and the result is displayed.

Please accept apologies for this inconvinience, but the plot file cannot be named after the session. Since multiple sessions can be plotted at once, this may lead to very long file names. Therefore, the name of the result contains the current date and time.

## 5 Examples

These examples are here to verify that the program works correctly.

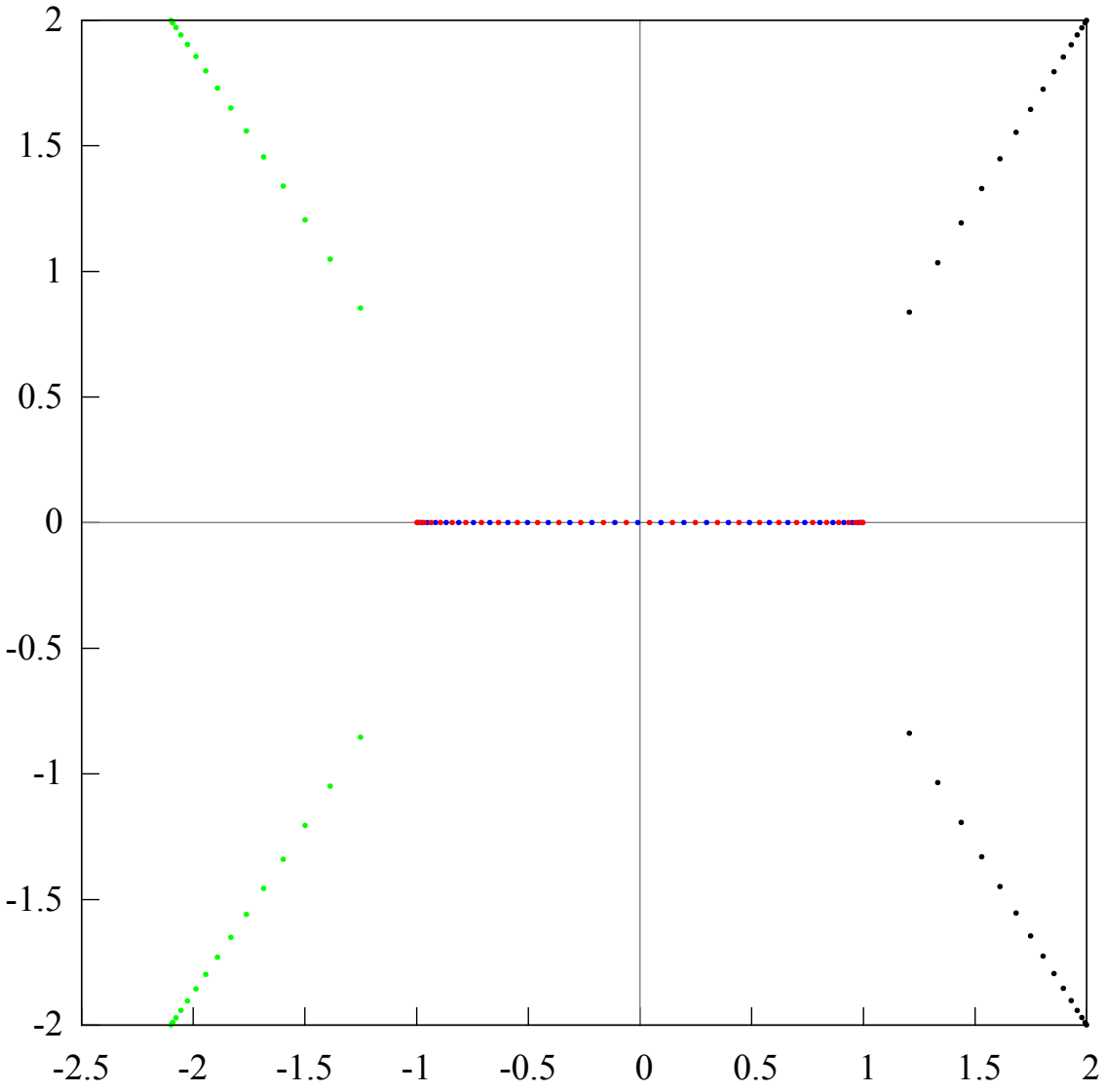
Example 1.
Start the program.
Press the Start button and wait 10 seconds.
Press the Create plot button. This graphic should be displayed.




We have used the functions:

```
funF = 1.0*z/sqrt(1.0-z^2)
funG = 1.0*z/sqrt((1.0-(2.0+I*2.0)*z)*(1.0-(2.0-I*2.0)*z))
```

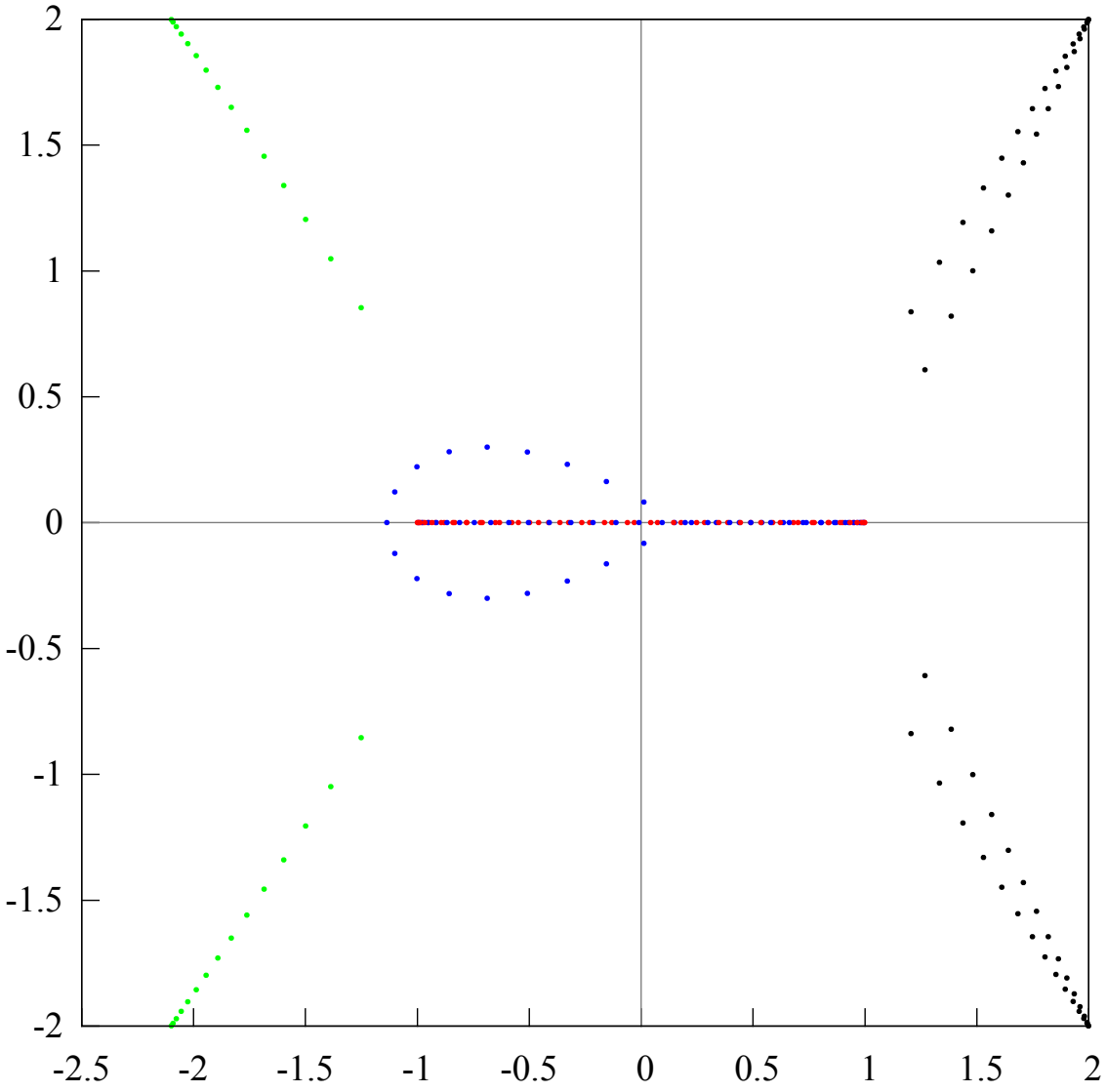
Example 2.
Start the program. Select the MHEPA approximant.
Press the Start button and wait 10 seconds.
Press the Create plot button. This graphic should be displayed.




We have used the functions:

```
funF = 1.0*z/sqrt(1.0-z^2)
funG = 1.0*z/sqrt((1.0-(2.0+I*2.0)*z)*(1.0-(2.0-I*2.0)*z))
funH = 1.0*z/sqrt((1.0-(-2.1+I*2.0)*z)*(1.0-(-2.1-I*2.0)*z))
```

Example 3.
Start the program.
Stay on the HEPA approximant, press the Start button and wait awhile.
Select the MHEPA approximant, press the Start button again.
There should be two sessions in the list.
Press the Create plot button. This graphic should be displayed.




The program should display these three graphics.
Otherwise, please check the installation instructions.

# 6 Errors

Note that the program works with `PARI/GP` versions 2.5.x and 2.7.x. Previous versions might work, but are not tested.

division by zero: Please change the functions and carefully check the input.

3 should divide valuation (= 1) in sqrtn: This happens when some part of a function does not have a Taylor expansion at all. For example $z^{1/3}f(z)$, even if $f(z)$ has a Taylor expansion.

polcoeff: non existent component in truecoeff: This happens when a function is divided by $z$, $z^2$, $z^3$, etc. Increase the Taylor expansion by 1, 2, 3, respectively.

matsolve: the PARI stack overflows: Please allocate 2000 MB of memory or more.

matsolve: not enough memory: The internal memory limit for `PARI/GP` 32-bit is around 1500 MB. Please edit `config.ini` and use `PARI/GP` 64-bit, which does not have memory limits.

gnuplot warnings: When all points are on the real axis, `gnuplot` automatically adjusts the imaginary axis to $[-1, 1]$. When a plot file is missing, `gnuplot` skips it.

# 7 Changelog

Changelog for the program HEPAComp:

- 1.0.0 (March 20, 2016)
  - Added list of sessions, allows plotting of several approximants at once
  - Restored context menu, added context menu for the list
  - Removed the fork MPPAComp, added approximant TPPA in HEPAComp
  - Added approximant Discriminant for HEPA (HEPAD)
  - Added approximant Special index for HEPA (HEPAS)
- 0.9.9 (December 31, 2014)
  - PARI/GP Scripts cleaned up and proofed
  - Removed approximant AHEPA (no functions applicable)
  - Removed menus, context menu (code cleanup)
  - Program is now cross-platform, see config.ini
- 0.9.7 (April 6, 2014)
  - Forked the program as MPPAComp
  - Added approximant TPPA in MPPAComp
- 0.9.5 (February 23, 2014)
  - Changed to matsolve in the algorithms, 50% faster than before
  - Added memory option for PARI/GP, matsolve needs a lot of memory
- 0.9 (February 18, 2014)
  - Added approximant AHEPA
  - Added list of working configurations
  - Added menus, context menu
  - Interface redesigned
- 0.7 (January 9, 2014)
  - Added approximant MHEPA
  - Interface streamlined
- 0.5 (December 15, 2013)
  - Initial version
  - Added approximant PADE
  - Added approximant HEPA

## 8 History

The most important aspects of the evolution of the algorithms:

- March 19, 2016 – added HEPAD and HEPAS
- December 28, 2014 – added "non-diagonal" version for PADE, HEPA, MHEPA
- April 2, 2014 – modified for TPPA
- February 23, 2014 – changed from manual matrix reduction to matsolve(), in effect removed the algorithm from February 23, 2013
- February 4, 2014 – polynomial size from polroots(): changed from fixed length to dynamic length with length(pts)
- January 30, 2014 – added polroots() for AHEPA
- January 27, 2014 – modified for AHEPA
- January 6, 2014 – modified for MHEPA
- December 3, 2013 – improvements for arrA, arrB
- November 27, 2013 – modified for Hermite-Padé
- November 22, 2013 – added polroots()
- February 23, 2013 – initial version

## 9 Abbreviations

Abbreviations used in the document:

- PADE: Padé approximant of $F(z)$
- HEPA: Hermite-Padé approximant of $[1, F, G]$
- MHEPA: Hermite-Padé approximant of $[1, F, G, H]$
- AHEPA: Hermite-Padé approximant of $[1, F, G, H, \ldots]$
- TPPA: Two-point Padé approximant of $F$ and $G$
- MPPA: Multipoint Padé approximant (not implemented)
- HEPAD: Discriminant for Hermite-Padé approximant of $[1, F, G]$
- HEPAS: Special index for Hermite-Padé approximant of $[1, F, G]$

## 10 Internet links

Internet links for the computer programs:

- PARI/GP 2.7.5 `http://pari.math.u-bordeaux.fr/`
- gnuplot 4.6.3 `http://gnuplot.info/`
- SumatraPDF 2.1 `http://www.sumatrapdfreader.org/`
- Eclipse Indigo SR2 3.7.2 `http://eclipse.org/indigo/`
- HEPA Computation 1.0.0 `http://justmathbg.info/hepacomp.html`