Computation of Hermite-Padé approximants and two-point Padé approximant (HEPAComp 0.9.9)

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1 PARI/GP options

This program computes Padé approximant of a function $F(z)$ and Hermite-Padé approximant of collecion of functions $[1, F(z), G(z)]$ (and also $[1, F(z), G(z), H(z)]$). It can compute two-point Padé approximant of $F(z)$ and $G(z)$.

The program reads "config.ini" from the current directory on startup. Input the desired paths to PARI/GP, gnuplot and a file viewer.

Please note that the program works with PARI/GP 2.5.x, 2.7.x and gnuplot 4.6.x. Previous versions of gnuplot might work, but has not been tested. Previous versions of PARI/GP work for the calculations, but there are specific problems with the "input/error streams", which are read by the program, in the Win32 versions (these problems might not be present in the Linux versions).

Press Start once and wait until it completes, that is the text "Competed in" appears. Then press Create plot. Select different options for the plot and then press Create plot again.

All files are placed in the subdirectory "temp". Most of them are plain text and can be opened with Notepad, Notepad++ and other text editors. The PDF file is binary and can be opened with Adobe Reader, Foxit Reader, SumatraPDF, PSView, Evince and other PDF viewers. The EPS file is plain text and can be opened with SumatraPDF, PSView and other PS viewers.

Open the log file to see everything about the current session.

Session name: It is determined automatically by the type of the approximant, the degre of the polynomials and four random digits. Select the checkbox to input the name. The name can contain digits 0-9, letters A-z, a-z, underscore, dash and parentheses (). This name is used for all files created, which are described later.
In both cases, the name is checked against all *.log files in the "temp" directory The name should be unique, otherwise the data files may contain several sets of points (since PARI/GP appends the information in the files).

Precision (digits): Significant decimal digits after the decimal point. This must be an integer number, at least 100. By default this is set to 1000.

Polynomial degree: "Diagonal" approximation: All polynomials (which are the result of the approximation) have this degree. Denote the degree by N. Only the first polynomial, named $P(z)$, may have $N-1$ or N zeros. The rest of the polynomials are guaranteed to have exactly N zeros. The degree must be an integer value, at least 1. By default this is set to 30.
"Non-diagonal" approximation: input several integer values separated by comma. Their count must be equal to the approximant's number of polynomials. PADE has 2 polynomials, HEPA - 3, MHEPA - 4.

Multiple approximations: Select the checkbox to set an integer number, greater than the initial degree, to compute sevaral approximations. This only works with "diagonal" apprximation. Number of 50 means that appoximants for $30, 31, 32, \ldots 50$ are computed. By default the checkbox is not selected.

Taylor expansion: Maximum order of the Taylor expansion. The formula is: add the degrees of the polynomials, the number of the polynomials and subtract 1. For "diagonal" approximation the formula is: $pol * N + pol - 1$. For example, let $N = 30$. PADE has 2 polynomials, then the maximal order is 61. HEPA has 3 polynomials, order is 92. MHEPA has 4 polynomials, order is 123.

Input this number only for the error "polcoeff: non existent component in truecoeff". This happens when the function is divided by z, z^2, z^3. Increase the number by 1, 2, 3, respectively.

Approximant type: Select between Padé approximant of $F(z)$, Hermite-Padé approximant of $[1, F, G]$, Hermite-Padé approximant of $[1, F, G, H]$.

The functions: Three functions can be typed in or selected by the dropdown menu. Their internal names are funF, funG and funH. Note that funF is evaluated first, then funG and finally funH. Multiplication by 1.0 forces PARI/GP to use real numbers (if no real numbers are present in the function). The imaginary unit is denoted by capital I.

After calculations: Select between staying at the origin and sending to the infinity point. By default the approximant is sent to the infinity point for Hermite-Padé approximants. And it is kept at the origin for two-point Padé approximant.

Transformations: These transformations are applied on every zero of every polynomial. Choose between Joukovsky transform and inverse Joukovsky transform. By default this is set to None.

Memory (in megabytes): Increase this for the error "the PARI stack overflows". By default this is set to 700.

Start: When the button is pressed, all data from the panel is collected and a script named pade30_1000_1234.gp is created. Then PARI/GP reads this script and creates data files with plot points, which are saved as pade30_1000_1234p.gpd, pade30_1000_1234q.gpd for the polynomials $P(z)$, $Q(z)$, respectively. These files have to be parsed for gnuplot, therefore the final data files are saved as pade30_1000_1234p.gpt, pade30_1000_1234q.gpt.

Please do not press Start again when a computation is running. There will be errors. Close the program and start over.

Stop: If a computation is in progress, press this button first before exiting the program. Otherwise the process "gp.exe" continues running in the background, then use the operating system's Task Manager to terminate it. This is the only usage of the button.

2 gnuplot options

Select session: Open a previous computation from here. The information about the session is saved on the first line of the log file.

Select polynomials: There are checkboxes for each polynomial, select the desired ones to plot.

Range of the plot: The range can be automatically determined by gnuplot or manually entered. The common ranges are there for convenience.

Range for X/Y axis: The values can be real and integer numbers.

Style of the plot: Choose between dots with color and symbols with color and without. The colors are blue, red, black, green for $P(z)$, $Q(z)$, $R(z)$, $S(z)$, respectively. The symbols are circle, filled circle, diamond, filled diamond for $P(z)$, $Q(z)$, $R(z)$, $S(z)$, respectively. The dots have size 0.125 (in gnuplot terms), while the rest have size 0.3 (much easier to see on the plot). By default the plot uses dots.

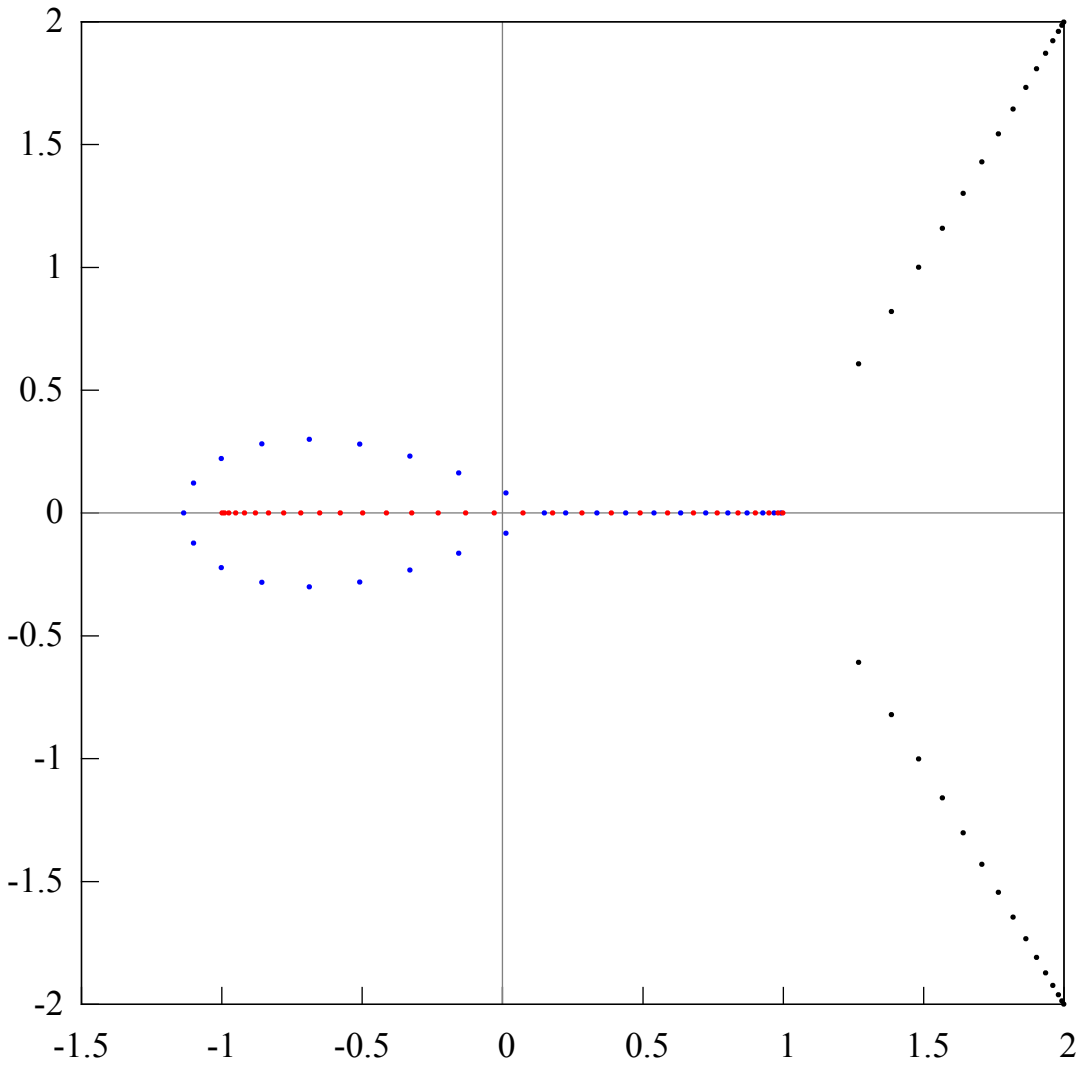
File type: Choose between Portable Document Format and Encapsulated PostScript. The gnuplot backend is "cairo graphics" and the respective terminals are pdfcairo and epscairo.

Create plot: When this button is pressed, all data from the panel is collected and a script named pade30_1000_1234.plt is created. Then gnuplot reads the script and creates the plot as pade30_1000_1234.pdf (or EPS). Finally, the file viewer is used to open the PDF/EPS file.

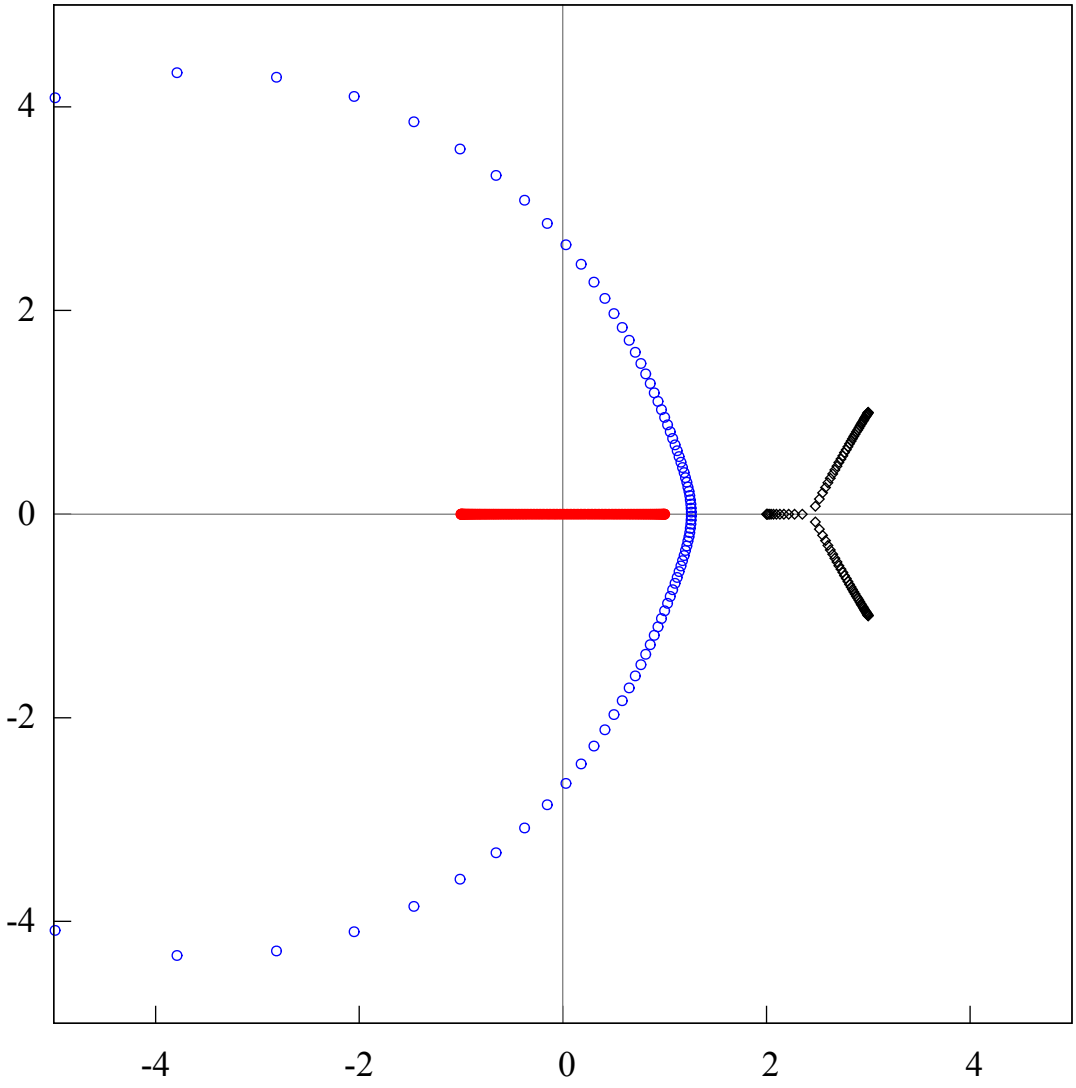
Once a session is available, a plot can be created many times – change the plot style and the plot range, select different combinations of polynomials, etc.

3 Examples

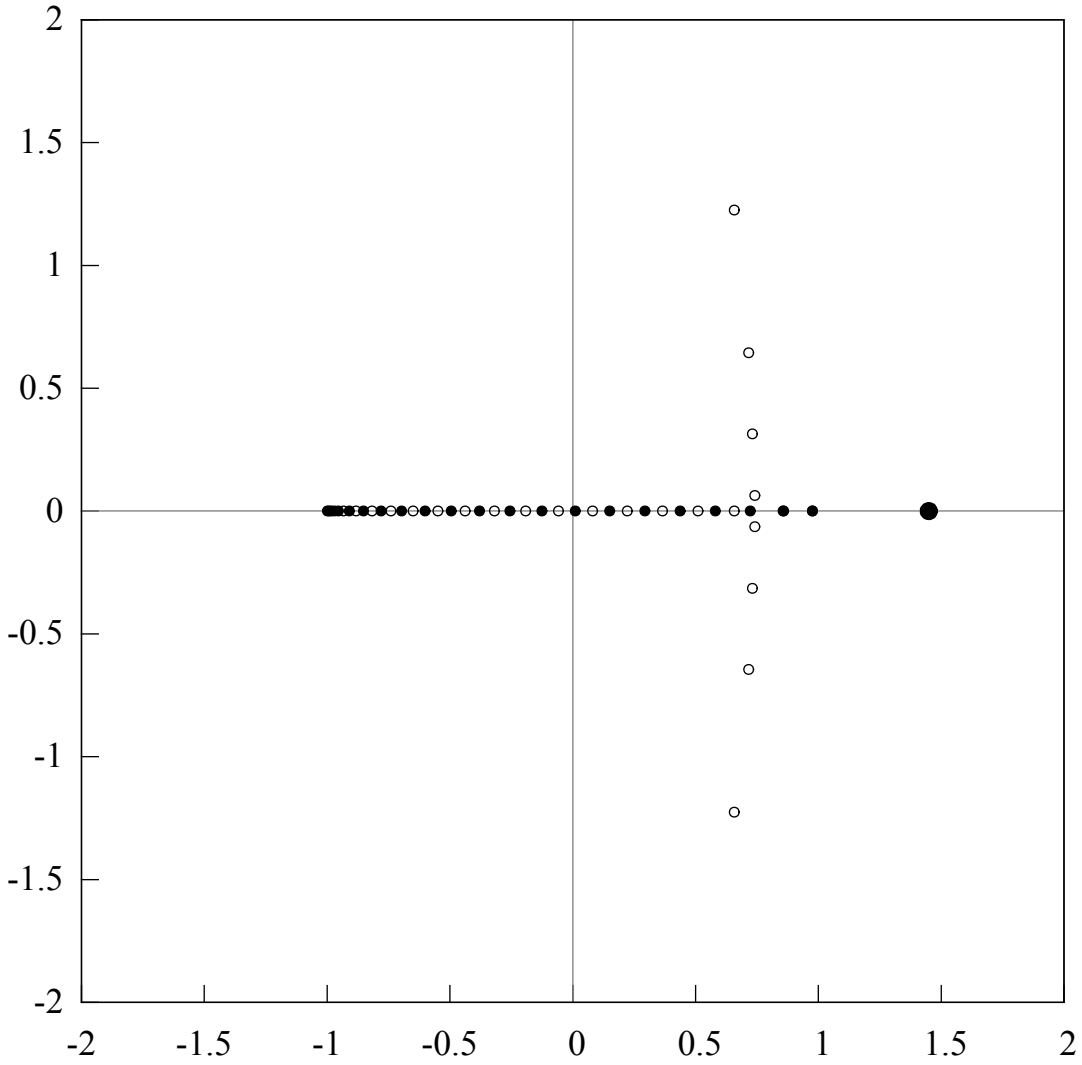
Example 1.
Press Start and wait 10 seconds.
Press Create plot. This graphic should be displayed.



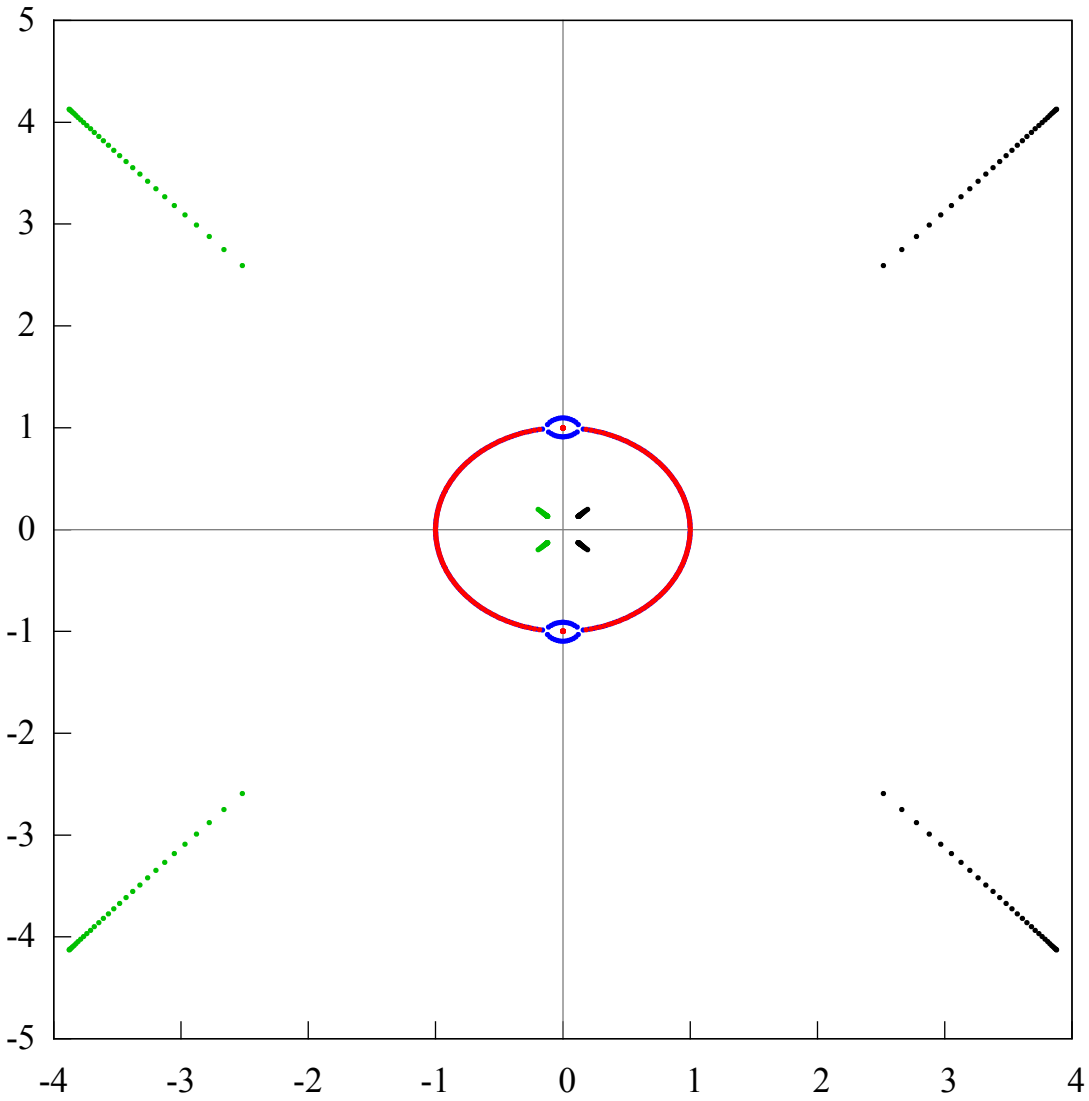
Example 2.
Input 90 into polynomial degree.
Select the third entry for funG: `1.0*((10*z^2-6*z+1)/(1-2*z)^2)^(1/3)`.
Press Start and wait a minute.
Select plot range $[-5, 5]$.
Select "symbols with color" for style of the plot.
Press Create plot. This graphic should be displayed.



Example 3.
Select Padé Approximant.
Select the second entry for funF: `1.0*z/sqrt(1.0-z^2)+1/(1.0-1.45*z)^10`.
Press Start and wait a second.
Select plot range $[-2, 2]$.
Select "symbols no color" for style of the plot.
Press Create plot. This graphic should be displayed.



Example 4.
Input 120,90,60,60 into polynomial degree.
Select Hermite-Padé Approximant of $[1, F, G, H]$.
Select inverse Joukovsky transform from transformations.
Press Start and wait a minute.
Press Create plot. This graphic should be displayed.



4 Errors

Note that the program works with PARI/GP versions 2.5.x and 2.7.x. Previous versions might work, but are not supported.

division by zero: this is exactly what it says, change the functions and it might work.

3 should divide valuation (= 1) in sqrtn: this happens when some part of a function does not have a Taylor expansion at all; for example $z^{1/3}f(z)$, even if $f(z)$ has a Taylor expansion.

polcoeff: non existent component in truecoeff: this happens when a function is divided by z, z^2, z^3, etc. Increase the Taylor expansion by 1, 2, 3, respectively.

matsolve: the PARI stack overflows: this happens when PARI/GP runs out of memory; increase the memory to 800 MB or more.

matsolve: not enough memory: this happens when some internal memory limit is reached, unrecoverable for now.

gnuplot warnings: when all points are on the real axis, it automatically adjusts the imaginary axis to $[-1, 1]$; when a plot file is missing, it is skipped.

5 History and changelog

History of the program

- 0.9.9 (December 31, 2014)
  - Scripts cleaned up and proofed
  - Removed AHEPA (no functions applicable)
  - Removed menus, context menus (code cleanup)
  - Program is now cross-platform, see config.ini
- 0.9.7 (April 6, 2014)
  - Forked the program as MPPAComp
  - Added TPPA in MPPAComp
- 0.9.5 (February 23, 2014)
  - Changed to matsolve() in the algorithms, 50% faster than before
  - Added memory option for PARI/GP, matsolve() needs a lot of memory
- 0.9 (February 18, 2014)
  - Added AHEPA, list of working configurations
  - Added menus, context menu
  - Interface redesigned
- 0.7 (January 9, 2014)
  - Added MHEPA
  - Interface streamlined
  - Has problems, use 0.5 or newer
- 0.5 (December 15, 2013)
  - Initial version, PADE and HEPA

History of the algorithms

- December 28, 2014 – added "non-diagonal" version for PADE, HEPA, MHEPA
- April 2, 2014 – modified for TPPA
- February 23, 2014 – changed from manual matrix reduction to matsolve(), in effect removed the algorithm from February 23, 2013
- February 4, 2014 – polynomial size from polroots(): changed from fixed length to dynamic length with length(pts)
- January 30, 2014 – added polroots() for AHEPA

- January 27, 2014 – modified for AHEPA
- January 6, 2014 – modified for MHEPA
- December 3, 2013 – improvements for arrA, arrB
- November 27, 2013 – modified for Hermite-Padé
- November 22, 2013 – added polroots()
- February 23, 2013 – initial version

Abbreviations

- PADE: Padé approximant of $F(z)$
- HEPA: Hermite-Padé approximant of $[1, F(z), G(z)]$
- MHEPA: Hermite-Padé approximant of $[1, F(z), G(z), H(z)]$
- AHEPA: Hermite-Padé approximant of $[1, F(z), G(z), H(z), \ldots]$
- TPPA: Two-point Padé approximant of $F(z)$ and $G(z)$
- MPPA: Multipoint Padé approximant

Internet links

- PARI/GP 2.5.5 `http://pari.math.u-bordeaux.fr/`
- gnuplot 4.6.3 `http://gnuplot.info/`
- SumatraPDF 2.1 `http://www.sumatrapdfreader.org/`
- Eclipse Indigo SR2 3.7.2 `http://eclipse.org/indigo/`

HEPAComp 0.9.9
`http://justmathbg.info/hepacomp.html`